

Chapter 6

Time Value of Money (TVM)

Chapter Objectives

Students must be able to:

- Explain the Concept of Time Value Of Money
- Identify the Major Components of Interest Rate
- Explain the Difference Between Simple Interest and Compound Interest
- Compute the Repayments In a Car Loan Using Simple Interest
- Explain the Rule Of 78 as Regards Interest Rebate
- Compute FV and PV of Single Sums
- Compute 'N' and 'i' in a TVM Problem
- Compute the PV of an Ordinary Annuity and an Annuity Due
- Solve Education and Retirement Funding Problems
- Compute FV of an Ordinary Annuity Like EPF Contributions
- Compute Housing Mortgage Payments
- Perform Amortization Calculations to Determine Interest and Principal Paid to Date and the Outstanding Principal Balance With the Bank
- Understand the Use of Inflation Adjusted Interest Rate
- Understand Compounding Frequency and the Effective Interest Rate
- Explain Uneven Cash Flows and the Use of Cash Flow Function In a Hand Held Calculator
- Understand Investment Appraisal Using NPV and IRR Calculations

Chapter 6

Time Value of Money (TVM)

Introduction

As I write this chapter the golden voice of Mohd Rafi is singing a Hindi song and I am suddenly reminded of the immortal value of great songs and movies. Time does not tarnish the value of these great achievements that produce a lasting human interest. However, time does tarnish the value of money. Money as a unit will retain its same unit value even after a hundred years although the look of money may have been changed. What causes it to lose value is the growth in the supply of money. There are more money lying around than used to be. Supply of money is constantly increasing and there are numerous ways it increases in supply. The Central Bank issues regularly new currencies and the banking system also promptly creates more money in the system through lending out more than 90% of its deposits. Money circulates and creates its own velocity. There is a need to delicately balance the supply of money to keep the general price level relatively stable. You need cash for transactions but it must be just enough to keep prices stable. If the government prints more money than the general price level will start increasing. Everyone who has been to Indonesia or the Philippines will wonder why the notes have such large denominations. It started somewhere and you now buy a loaf of bread with a 1,000 currency note. But this is a chapter on time value of money (TVM). The inflationary trend has a bearing on future values of money. In fact one of the three main components that reflect the price of money is the inflation expectation. But let us start understanding the mechanics of time value of money with the basics.

Money Today versus Money Tomorrow

We have to get to first base. Would you rather be holding money now versus tomorrow? The obvious answer would be yes I want the money now. Why? You may say, "Because I can spend it now and enjoy the purchase." There is a preference to spend now, the current consumption appeal. You can get immediate satisfaction. Or you can invest that money today and it can grow to more money in the future. There is an opportunity for you to invest the money. Or you might say it is safer if I hold it now because you might lose it or you might not give me back. These simple reasons give value to money now versus tomorrow.

The issue is what would make you give that money to me to use. You have to forgo current consumption and so would want some compensation for it. "Pay me ten percent more after one year" you might say. This value for delaying or postponing consumption is called the pure time preference value of money. You will also immediately realize that it is not only consumption that

you forgo but also the investment opportunity. So for forgoing the investment opportunity you may want another five percent because that is the rate you would have earned from depositing it in your neighborhood bank. And you realize that this person is planning to invest your money in a business. You start wondering if you will get your money back. For the slight palpitation in risk taking you want another five percent. We now have an individual who has a time value of 20% for his money.

On further thinking you might realize that the value of money is being eroded by inflation. The longer you wait to spend the money the less it can buy things in the future. You then expect to be compensated for this inflation factor. You want another 3%. Now the individual is looking at 23% for his money.

Now, if we add all the individuals in the community, we will soon get an average rate for the time value of money. It may not be 23% for all. For those who have a lot of spare money their consumption, pure time preference value, may just be 3%. So they will be satisfied with 16%. There are many more with different interest values. You then figure that the price I get paid for lending my money also depends on the demand for my money. Perhaps there are not too many takers for my idle money. I may have to be satisfied with 9% and thus the aggregate demand and supply will determine the price of money. You are of course wondering, am I a price maker or price taker. The man on the street is quite powerful. Collectively they can influence the price of money, what we call the interest rate. But with the aggregation of all players we do not see the simple tussle between demand and supply for money. We simply accept the market rate as an equilibrium price for all the three or four factors that influences the price of money:

Price of Money or Interest Rate

1. Pure time preference
2. Inflation
3. Investment opportunity cost
4. Risk premium

It is difficult to exactly pin point how much of an interest rate is for pure time preference, inflation expectation, investment opportunity cost and risk premium. We are only aware that all these factors affect it. The people on the supply side will balance off with the people on the demand side.

When we tried to piece together an interest rate we were looking at quite high interest rates but in reality the interest rate you get is quite low. The banks are offering you right now less than 4% for your money. Inflation is going at 4% on average. You accept that there is less risk when you give your money to the bank. But is 4% enough compensation for your delayed gratification. The answer is a resounding no. The government knows it and often issues savings bonds to retirees that offer a guaranteed 5% rate of interest. There are not enough opportunities to make our money grow higher without taking more risk. Therefore we invest in more risky assets like stocks or real estate to derive a higher return. We therefore see the interplay of risk coming into the picture when we talk about investing our money. These are the factors in the economy which make this subject confusing but interesting. These introductory comments relate to macroeconomics and will be the

subject of attention of economists. For us in this module we explore the mechanics of how these rates affect our financial decision making.

The Components of Interest Rate

In various literatures they have narrowed down three factors as affecting the interest rate:

1. the pure time preference
2. inflation
3. risk

Conventionally we have different names for the impact of these three factors. The rate that reflects the pure time preference is reflected by the long term economic growth rate. The economy is offering this economic opportunity cost. The inflation expectation is added to this pure time preference to give you the risk free rate. Money supply or inflation is going along in Malaysia at about three percent. The risk free rate therefore includes the pure time preference and inflation. When we add the risk premium to the risk free rate we have the nominal interest rate. So the stated interest rate is the nominal interest rate. When we take out the risk factor we have the risk free rate. This rate is associated with government securities as the government securities like Treasury bills are considered very safe. When the nominal interest rate is adjusted for inflation it is called the 'real' rate of interest. The word "real" indicates that the inflation factor is taken out. This means that the real growth rate or improvement in economic value is caused by the pure time preference and risk premium.

Accountants in order to arrive at the real interest rate generally adjust the nominal interest rate by deducting the inflation factor. In time value of money calculations the adjustment will require a further division by (1+ the inflation factor).

For example: If the stated nominal interest rate is 10 percent and the inflation rate is 4%, the real rate is $10\% - 4\% = 6\%$. For TVM calculations we use $10\% - 4\% / 1 + .04 = 5.7692\%$. There is a slight difference.

It is important to look at real rates because inflation eats into your purchasing power. Because cost of goods have gone up by say 4% your actual growth in buying power is $10\% - 4\% = 6\%$. Inflation of prices has eaten into your growth in money terms.

The risk premium is an amalgamation of many factors. There is business risk, financial risk, market risk, liquidity risk, individual asset risk, exchange rate currency risk and a few others. These risks work together to give an appropriate risk premium for each asset class.

In this module we will be content to take the interest rate as given and work with an assumed rate for inflation, risk or investment.

Simple Interest versus Compound Interest

There are two types of interest, simple interest and compound interest. Simple interest is defined as the interest rate that is applied to the original principal sum. Each year the interest rate is applied to the original principal amount. Whereas the compound interest is computed by applying the interest rate to the compound of the principal and interest earned in the previous periods. Interest earned is added to the principal and so the amount of interest earned increases at an increasing rate. As years go on the compound interest will increase dramatically. The difference between simple interest and compound interest is simply whether we add the interest earned or not. Interest on interest makes the difference in amount earned

Simple interest formula is:

Future Value (FV) = Present Value (PV) times [1+ (rate of interest * time (t))]

In short, $FV = PV (1+rt)$

Simple interest is used to calculate future values and present values in money market instruments. The simple interest is also used in hire-purchase car loans.

Example of Car Loan

When you buy a car under hire-purchase you will make a down payment and borrow the difference and pay the interest and principal loaned in fixed monthly installments. For e.g. if the car costs RM 140,000 and you make a down payment of RM 20,000 you are effectively borrowing RM 120,000 from the hire-purchase company. If say the interest charged is 4% and the period of hire purchase is 5 years the monthly installments are calculated as follows:

Amount borrowed = RM 120,000

Interest charged is 4% of RM 120,000 for each year = RM 4,800

Total interest charged is 5 years * RM 4,800 = RM 24,000.

The total interest is added to the principal loan to give RM 144,000

RM 144,000 divided by the number of months (60) = RM 2,400 monthly payment.

The formula is $FV = PV (1+rt)$

Therefore, $PV = 120,000$, the amount borrowed.

The (r) is 4% and (t) is 5.

$FV = 120,000 [1 + (0.04*5)]$

= 120,000 * 1.20

= 144,000.

The 144,000 is the full payment and when divided by the number of months gives you a monthly payment of RM 2,400.

You will notice that the interest rate 4% is charged on the original loan amount of 120,000 for all the 5 years.

Simple Interest applied to Money Market Instrument

In the money market the instruments are bought and sold using simple interest. The maturity value is the FV and the buying price is the PV. The present value is a discounted value. That is why these investment securities are called discounted securities. The formula is simple interest formula.

Say you are buying a 90 days treasury bill that offers an 8% annual interest yield. The future value is known because they have a fixed future value. Let us say it is RM 1,000. We can now use the formula to calculate the PV or the buying price.

$$FV = PV [1+rt]$$

$1,000 = PV [1 + (0.08 \times 90 / 365)]$. The (t) is 90 days divided by 365 days because the interest earning period is only 90 days out of the 365 days in the year. To find the PV, the buying price we divide the FV by the product of $[1+rt]$.

$$\begin{aligned} \text{Therefore, } PV &= 1000 / 1.0197 \\ &= 980.68 \end{aligned}$$

You will therefore pay RM 980.68 for the Treasury bill and earn the difference i.e. 1000 less 980.68 = RM 19.32 in interest for the 90 days. The interest is always quoted in annual terms i.e. 8% per annum. But the (t) will be adjusted for the actual number of days out of 365 days it is earning interest.

Compound Interest

In calculating future values we generally use compounding because it is assumed that investors will re-invest their interest into the account. The interest earned in the previous period is re-invested. In this way the future value is compounded to a much larger value.

This is a big assumption as investors in general may withdraw their interest or dividends to spend each year. This is their income from investment. In the unit trust industry this is a big issue. Do investors take their interest or dividends out and spend it or re-invest it. If they re-invest their dividends or distributions they will derive the compound effect. If not, they will only get the simple interest effect. Many reading this paragraph will feel guilty spending their dividends and interest instead of re-investing it.

To understand this impact we will have to go through the compound interest formula.

There are only five variables in compound interest calculations. They are:

PV = Present Value

FV = Future Value

N = number of compounding periods

i = annual interest rate applicable

Pmt = payment per period

With these five variables we solve a lot of financial planning issues. Fortunately we do not have to do all the calculations in our head. We use the financial calculator. Therefore to understand this chapter you will need a financial calculator. I am using the Hewlett Packard 10BII. Many students use the Casio FC 100 or 200 which has a menu format. Some will use the Texas Instrument BAII plus. You need to purchase any one of these calculators and understand the many key or input functions. Each of these calculators has different key strokes. In this chapter we will follow the theory of the calculation and you will have to learn how to input these variables into your personal financial calculator.

The Usual Calculations in Time Value of Money (TVM)

1. Future value of a single sum
2. Present value of a single sum
3. Number of years or compounding periods
4. The interest rate earned per year or per compounding period
5. Present value of an Ordinary Annuity
6. Present value of an Annuity Due
7. Future value of an Ordinary Annuity
8. Future value of an Annuity Due
9. Periodic payments

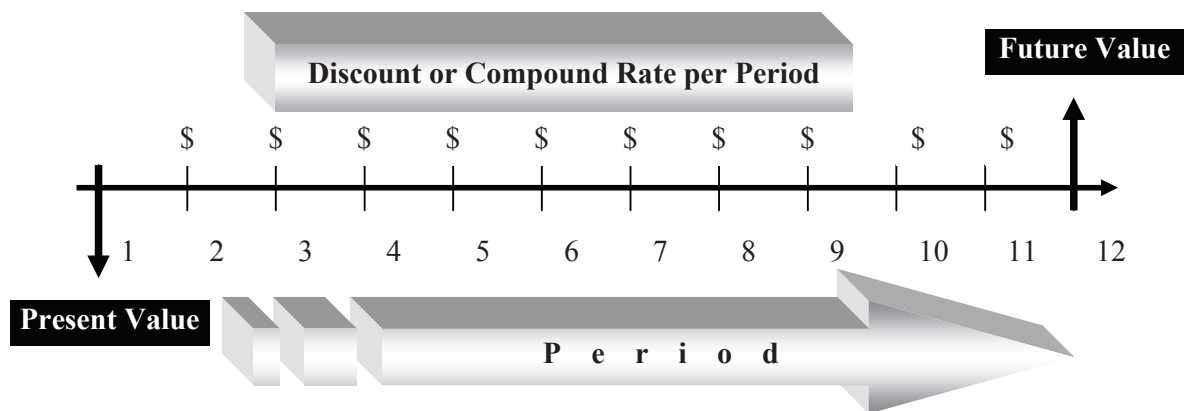
We first start with the **formula for compound interest**:

$$FV = PV (1 + i)^n$$

Read as Future Value = Present Value * One plus the interest rate to the power of (n).

When working with percentages in a calculation we will have to convert them to decimals. It will also be good to draw a Time Line for each situation.

Time-Line Illustration of Relationship between *Future Value* and *Present Value*



Future Value of a Single Sum

To find the Future value of a single sum of RM 10,000 placed in an investment that earns 8% a year and left to compound for 10 years will give the following future value:

$$FV = PV (1+i)^n$$

$$\begin{aligned} FV &= 10,000 (1+0.08)^n \\ &= 10,000 (2.1589) \\ &= 21,589 \end{aligned}$$

The trick is now to use the calculator to get the same answer. Different calculators use different key strokes. In the HP 10BII I will use the following steps.

Step 1: State the number of payments or compounding per year. In this case it is annual compounding and therefore the number of payments or compounding is one per year.

Step 2: I will enter the present value of 10,000. In the calculator one variable entered must be negative. It is customary to put the negative value to the Present Value or Payments. An explanation for that is that present values are usually your investment or cost and therefore money is being put to work. Payments are the same. Therefore these values are placed as a negative value.

Step 3: I will enter the (n), the number of years or compounding periods. In this case it is 10 years. Since the compounding happens once a year $N=10$. If compounding were to happen each month then $n = 12 * 10 = 120$. In step one I would have used 12 as the number of compounding in a year.

Step 4: I will enter the interest rate. Since the compounding is once a year (i) is expressed as the annual percentage of 8%. If the compounding happens once a month then the interest rate is divided by 12 in most calculators. In the HP 10BII there is no need to divide the interest by 12 as the initial entry of how many payments or compounding will divide it automatically.

Step 5: I will now simply press for Future Value. The answer appears as RM 21,589.

For most financial calculations, there are three known variables and we just need to find the missing variable. In this example we knew the PV, the i, and n. We needed to find the fourth variable, the FV.

In Casio FC 100 and 200 there are menu buttons. We must first press the function mode of CMPD to state that we are doing TVM calculations. There are some setting functions to perform first. We then use the scroll key to enter the various variables. We still have to use a negative sign for PV or Pmt. After entering the known variables we press “solve” to get the answer. In Casio we need to calculate the number of periods of compounding. Therefore if the compounding is monthly the n entered will be $10 * 12$. For (i) we will divide the annual interest rate by 12 to give a monthly interest rate.

To complicate matters we have the old version of Casio 100 and 200 and the new menu driven version. Students bring along all types, old and new to the class and without the manual. HP 10BII is pretty easy to use. But people who need a menu to prompt them may find the new Casio useful.

Present Value of a Single Sum

The second situation is to find the Present Value of a lump sum receivable in a future period. We want to know the present value of this future payment. The formula does not change. We solve for PV instead of FV. Therefore the

$$PV = FV / (1+i)^n$$

Example: What is PV of a future lump sum RM 10,000 receivable in 10 years? The theory is the money receivable in the future is not of the same value as money received today. The reasons have been explored before under the section on components of interest rates. The question now is how to discount this future sum. We need to use a discount rate. The discount rate is the reverse of the compound rate. It is the rate used to discount future cash flows to Present Value. It usually is the opportunity cost component. If a person has the money today how much interest or return will he earn each year? That is his opportunity cost. We use that rate as the discount rate. Let us assume the rate is 8%. Therefore, the PV will be:

$$\begin{aligned} PV &= 10,000 / (1.08)^n \text{ where } n = 10. \\ PV &= 10,000 / 2.1589 \\ &= 4,631.93 \end{aligned}$$

Using the financial calculator we input the FV 10,000, the n=10, and discount rate of 8%. We solve for PV and we will get 4,631.93. The calculator will show a negative -4,631.93. One variable will be negative. The PV or Pmt are used as the negative values.

To Find 'N', The Number of Years

Number of compounding periods or years

Another situation is knowing how long it is going to take to achieve a lump sum objective given a present sum investment. For example you have target lump sum to achieve of RM 10,000. You have currently in hand RM 3,000. You are able to earn 8% per annum. How many years will you take to reach the target?

Using the formula you solve for n.

$$FV = PV (1+i)^n$$

$$10,000 = 3,000 (1.08)^n$$

Solving for n requires the use of logarithms which is best left for a math class. However using the calculator we just have to input the three known variables to find out the unknown variable.

We know FV, PV, and i. WE solve for n.

The answer is 15.6439 years.

That is a very long time to wait despite earning 8% interest. The better option is to find extra work to earn the balance 7,000. In real life situations finding income seems a better option than forcing your investment ringgit to work harder. But clearly both count in your effort at increasing wealth.

Finding the Rate of Return or Interest Rate (The 'i' Factor).

There are often situations where you want to find out the compound interest rate earned. You know the current investment and you know the future value after 'n' number of years. You want to know what the compound rate of interest was.

For example: You have put in RM 5,000 into the investment. The fund offers you a target capital sum of RM 10,000 after 5 years. You wonder what the compound interest rate is. Plug in the inputs into the calculator; FV, PV and n=5 years. Solve for i.

You will get 14.8698%. That is a very good rate.

Short cut rule of 72

A short cut to calculate how long it takes or what is the rate required for a sum of money to double is by using the **Rule of 72**. To know the rate required to double RM 5,000 to RM 10,000 we divide 72 by the 5 years and you will get the answer as 14.40, a very close approximate. Similarly if you know you can earn 10%p.a. how long will it take to double your money? Divide 72 by 10 and you will get 7.2 years.

Future Value of Single Sum where Compounding Frequency is More than One

When the compounding frequency is more than 1, the formula for *future value of a single sum* $FV = PV (1 + i)^n$ is modified to become:

$$FV = PV (1 + i/m)^{m \times n}$$

The trick here when using the calculator is to multiply 'n' number of years by the frequency of compounding and divide the annual interest rate by the frequency of compounding.

Present Value of an Ordinary Annuity.

An annuity is a stream of equal periodic payments received at uniform intervals. You will see this annuity feature for example in your regular paychecks, your car payment, mortgage payment and even your EPF contributions.

The expression ordinary annuity means the payment is received or made at the end of the compounding period. Mortgage payments are made at the end of the month after receiving your salary or income. Your EPF contributions are made at the time your salary is received which is at the end of the salary month.

For example: If someone asks you to choose between a lump sum today of RM 100,000 or an ordinary annuity of RM 10,000 for the next 15 years, which would you choose. You know you can invest any money at 8% per annum. This is an ordinary annuity and therefore you receive them at the end of the year.

The variables you know are: Payment 10,000, n=15, and i=8%. We solve for PV and we get 85,595 (rounded to the nearest ringgit). The better choice is to receive the 100,000 up front. You would input the end mode for payments in the calculator. The default mode normally is the end mode.

The mathematical formula for **present value of ordinary annuity** may be expressed as:

$$PVOA = \frac{PMT \{1 - (1 + i)^{-n}\}}{(i)}$$

Where,

PVOA = Present value of an ordinary annuity

PMT = the amount of payment at regular
time interval

i = the yield per annum or the annual discounted rate

n = the time period required for discounting

Present Value of an Annuity Due

This is the same as the example above but this time the payments are received or made at the beginning of the period. In the example above you had to make a decision between receiving an ordinary annuity of 10,000 for 15 years versus a lump sum of 100,000. What if the 10,000 are receivable in the beginning of the period? Would you change your mind? Let us plug in the figures. The variables we know are:

Payment (beginning mode) 10,000,

N= 15 years

i= 8%

PV =?

Answer PV = 92,442. It is quite close to the offer of 100,000.

This is a very common situation in financial planning. In education funding we have to pay college fees at the beginning of each semester. In retirement you would also like to receive your retirement income at the beginning of each month as your expenses are already incurred from the beginning. Therefore to know how much capital to accumulate you find the Present Value of the annuity or payment.

The mathematical formula for **present value of annuity due** may be expressed as:

$$PVAD = \frac{PMT (1 + i) \{1 - (1 + i)^{-n}\}}{(i)}$$

Where,

PVAD = Present value of an annuity due

PMT = the amount of payment at regular
time interval

i = the yield per annum or the annual discounted rate

n = the time period required for discounting

Education Funding Example

For example: you have to pay college fees at the beginning of each semester. Each semester is 6 months. The fees are RM 10,000 each semester. There are 6 semesters. How much must you accumulate at the beginning of college to meet these fees? You realize that you can earn 8%p.a. on the college education fund that you have accumulated.

The answer is to find the Present Value of the college fees (annuity due) with $i=8\%$ p.a., $n=6$ periods with payment required of 10,000 per period. The only difficulty is the interest rate is for one year or 12 months. N is a period of 6 months each. Therefore we have to convert the 8% interest rate to a 6 months interest rate. We divide 8% by 2 to get the six month interest rate of 4%.

Pmt = 10,000 (beginning mode)

N = 6

$i = 4\%$

PV = ?

Answer PV = 54,518

In the HP 10BII I could have told the calculator that there are two payments per year. I then input the 8% per year interest rate. The number of periods is derived by 3 years times two. I have to tell the calculator that these are beginning mode payments.

Retirement Planning Example

Another common example is the lump sum required for retirement. To solve this example we need to determine the amount of retirement income desired by the individual. This would be an annuity due. We need to determine the number of periods for which this payment is to be received. We need to estimate his life expectancy. During this period his retirement fund will also be able to earn interest or a rate of return from the retirement funds.

For example, let us say the individual desires an income (annuity due) of 50,000 a year. Let us say he expects his life expectancy to be 25 years from his retirement. His investment rate of return is 8% p.a. How much must he accumulate for his retirement?

The variables we know are:

Pmt = 50,000 (beginning mode)

N = 25 years

$i = 8\%$

Solve for PV?

The answer is PV = 576,438 (rounded to nearest ringgit)

You can easily convert this example to a monthly payment schedule.

In this case $n=25$ years become $n=12*25 = 300$

The $i=8\%$ p.a. can be converted to monthly interest rate of $8\%/12 = 0.6667\%$ per month.

The payment of 50,000 a year can be converted to $50,000/12 = 4,167$ per month.

We then solve for Present Value.

The answer is PV = 543,451

You can see that when the compounding is monthly you will earn more. Therefore the present value required is much lower, a difference of RM 32,986.

Future Value of an Ordinary Annuity.

This is also a common financial planning calculation. The best example of this is our EPF contribution. We contribute a regular payment at the end of each month to the Employee Provident Fund. We will want to know the Future Value of these regular payments to see if we have enough money for retirement. The only complication here is our contributions change as our salaries change and the rate of return that EPF offers on our savings also changes often enough. For these reasons we have to make some assumptions.

The relevant formula for computing the FV of an ordinary annuity is:

$$FVOA = \frac{PMT [(1+i)^n - 1]}{(i)}$$

Where,

FVOA = future value of an ordinary annuity

PMT = the constant amount of payments at regular
time interval

i = the yield per annum or the annual compounded rate of
return on investment

n = the time period required for compounding

EPF Accumulation Example

Let us say your salary is RM 5,000 a month. Your annual income is therefore RM 60,000. Your EPF contribution would be 11% plus 12% = 23% of salary. Therefore your annual contribution is RM 13,800. Let us say you have 20 years more to retire. Let us further assume that EPF will continue to offer a 5%p.a. rate of return.

Let us now input the known variables.

Payment (end mode) = 13,800

N=20 years

i=5%

To solve for FV.

Answer is FV = 456,310.

The complications

What if after 10 years your salary increases to RM 8,000 a month. The best way to solve the future value is to split the contributions into two periods with the different salaries.

For example for 10 years your contribution is 13,800. You find the FV of these contributions. Then your contributions change to $8,000 * 12 * 23\% = 22,080$.

We may need to assume that EPF continues to give a 5% rate of return per annum.

Payment: 13,800 (end mode)

N=10 years

i=5%

FV = 173,575

This FV will now become the Present Value and we find the FV of this single sum given 5% (i) and (n) = 10. Therefore,

PV = 173,575

i= 5%

n=10

Solve Fv =?

Answer FV = 282,735

Then we find the future value of the new EPF contributions of RM 22,080 for 10 years and at 5% rate of return.

Payment = 22,080 (end mode)

i=5%

n=10

Solve for Fv?

FV = 277,720 (rounded to nearest ringgit)

The total EPF will now be $RM\ 282,735 + RM\ 277,720 = RM\ 560,455$.

Another complication would be the current savings in EPF. We can find the future value of this current amount and add to the future value of the annual payments. For example let us assume the current savings in the EPF is 100,000. We input the variables:

PV = 100,000 (negative sign)

i=5%

n=20 years

Solve for FV?

FV = 265,329

The total FV at EPF will now be:

$$\text{RM } 265,329 + \text{RM } 282,735 + \text{RM } 277,720 = 825,784.$$

A person earning 5,000 and 8,000 a month will have a reasonable end balance in the EPF. But unfortunately the average EPF lump sum withdrawn according to EPF sources is about RM 100,000 and they are hoping to make the average withdrawal equal RM 120,000.

Now that you can use the financial calculator you can find out that with this Present Value of RM 120,000 how much income you can have every month if you lived for another 25 years. We have to assume a rate of return on your Retirement fund. Let us say 6%.

We solve for Payment per month beginning mode.

$$\text{PV} = 120,000 \text{ (negative sign)}$$

$$\text{N} = 25 * 12 = 300$$

$$i = 6\% \text{ p.a.} / 12$$

Solve for Payment?

$$\text{Payment} = 769 \text{ per month.}$$

This must be a huge drop in living standards. With inflation rearing its ugly head it will be tough to live within 769 a month. One hopes the individual has other investment assets that generate income and that all his liabilities at least the main ones like mortgage and children's education have been settled.

Future Value of an Annuity Due.

With the EPF example we assumed that the annuity or payment was made at the end of the period. We can just as easily calculate the Future Value of an Annuity Due. A common example will be your regular savings each month or year into a fund. If you deposited in the beginning as opposed to end of the period you will earn an extra compounding effect. Let us take an example.

Your regular Payment is say RM 1,000 per month. Let us assume it is at the beginning of a month. You invest for 10 years. The average rate of return is say 8%p.a. You want to solve for Future Value. Enter the known variables:

$$\text{Payment (beginning mode)} = 1,000 \text{ (negative sign)}$$

$$\text{N} = 10 * 12 = 120$$

$$i = 8\% \text{ p.a.} / 12$$

Solve for FV?

$$\text{FV} = 184,166 \text{ (nearest ringgit)}$$

If on the other hand the payment had been made at the end of the period we will have

FV = 182,946. The difference is RM 1,220.

Mathematically we can derive the end mode figure by dividing the FV beginning mode by (1+i). We have been using the monthly interest rate of 0.6667 %. Therefore $184,166 / (1+0.0067) = 182,940$. There will be slight rounding off error.

The relevant formula for solving FV of an annuity due is:

$$FVAD = \frac{PMT (1 + i)[(1 + i)^n - 1]}{(i)}$$

Where,

FVAD = future value of an annuity due

PMT = the constant amount of payments at regular
time interval

i = the yield per annum or the annual compounded rate of
return on investment

n = the time period required for compounding

Periodic Payments

We have many situations that require us to calculate the periodic payments. The best example of periodic payments is your monthly housing mortgage payments.

Housing Mortgage example

A housing mortgage uses the same formula as the present value of an ordinary annuity. In this case you know the Present value of the Ordinary Annuity. It is the loan amount taken from the bank. Your task is to find the monthly mortgage payment.

Let us say you are buying a house costing RM 250,000. Your down payment is 10% and you borrow the difference of RM 225,000. You take a 20 year loan with monthly rest at an interest rate of 8% p.a.

You input the following known variables:

PV = 225,000

N= 20 *12

i= 8% / 12

You solve for Payment (end mode).

The Payment is RM 1,882 (rounded to the nearest ringgit)

You can leave the existing data in the calculator and experiment with various scenarios.

For example what would be the monthly payment if you took a longer term of 25 years? You just have to change (n) to $25 * 12$. The new payment will be RM 1,737. What about 30 years? The new payment will be RM 1,651. What do you notice? Increasing the term does not really bring down the monthly payment. That is why many who wanted two generation housing loans did not know TVM.

The 30 year loan would have resulted in a total payment of $30 * 12 * 1650 = 594,000$. The original term of 20 years will require a total payment of $20 * 12 * 1882 = 451,680$, a difference in total payment of RM 142,320. That difference will make a nice estate for the next of kin. The number of people high up in Malaysian society who suggested this 2-generation loan reveals the extent of financial illiteracy in the country. Or simply people who do not know how to use the financial calculator.

Amortization

With the housing loan mortgage we must realize that with each payment there is an interest portion and a principal portion. In the initial years the interest portion of the mortgage payment will be high. In the later years as the principal is slowly brought down the principal portion of the loan payment increases. With the financial calculator it is easy to calculate the interest and principal portion either of one mortgage payment or the sum total of interest and principal paid over a stated period.

We must first enter all the details for a periodic payment calculation. If we assume the data as per the 20 year mortgage loan of 225,000 at 8% per annum interest we can find the following.

After 10 years how much interest and principal have you paid and what is the principal balance owing to the bank.

There is an amortization function key. On the HP 10BII I will enter the mortgage details and then press: 1input 120 and then press Shift Amort. I will get the following figures by pressing the = sign.

Sum of Principal paid to date = 69,883.

Sum of interest paid to date = 155,955

Principal balance outstanding = 155,116. If you add this to the principal paid so far you will get the original loan of 225,000.

In the Casio FC 100 and 200, after entering the compound interest data for the housing loan you will press the Amort key and state the Payment 1 = 1 and payment 2 = 120. Then you solve for Sum of Interest paid, sum of principal paid and the balance outstanding.

Often times there will be reason to settle the housing loan in full. You will need to know how much the principal balance is to settle with the bank. Further when you are constructing the Net Worth statement you will want to enter the actual liability outstanding with the bank.

Inflation-adjusted Interest Rate

Inflation is a very hot topic today. In financial planning we have to provide for inflation in our calculations. The most critical situation is retirement income planning. If you are 20 years away from retirement your estimate of retirement income must take inflation into account. People will usually think in current ringgit terms. Therefore if someone says he is comfortable with RM 5,000 a month and he has another 15 years to retire, you will input this inflation factor to his retirement income. The TVM calculation will be:

$$PV = 5,000$$

$$N = 15 \text{ years}$$

$$I \text{ (inflation rate)} = \text{say } 4\%$$

Solve for FV?

$$FV = 9,004$$

This means because of inflation the required retirement income is not 5,000 a month but rather 9,000 a month.

The next issue is will he need to adjust for inflation in his retirement years. The answer is usually yes. Inflation will eat into your purchasing power and it will be prudent to provide for the impact of inflation. How do we do that? It is a simple task of adjusting the investment rate for this inflation factor. If the investment rate is 8%p.a. and inflation is expected to be 4% per annum the i-factor we enter in the TVM calculations is adjusted for inflation as follows:

$$R - I / (1+I)$$

Read as: **rate of investment less the inflation rate divided by (1 plus the inflation rate).**

$$8\% - 4\% / 1.04 = 3.8462\%$$

If we use all decimals:

$$0.08 - 0.04 / 1.04 = 0.038462 * 100 = 3.8462\%$$

Inflation-Adjusted Investment Rate- Retirement Example

Let us say the client wants RM 9,000 retirement income per month. His life expectancy is say 20 years. He wants his income to increase every year by 4%. His investment performance is 8% per annum. How much Present Value sum must he accumulate for his retirement?

Note: So far in the examples the compounding frequency and the payment frequency have been kept the same. In other words if the payment is monthly the compounding is also monthly. It is beyond the scope of this module to use a different payment and compounding

frequency. Therefore to stay with the simple assumption we will multiply the monthly income desired of RM 9,000 by 12 to give the annual retirement income of RM 108,000.

Let us input the known variables.

Payment (beginning mode) = 108,000 p.a.

N= 20

Inflation-Adjusted 'i' = $0.08 - 0.04/1.04 = 3.8462\% / 12 = 0.3205\%$ per month

Solve for PV?

PV = 1,545,184 (nearest ringgit)

If we did not adjust for inflation the Present Value required will be:

Payment (beginning mode) = 108,000 p.a.

N=20

i=8% / 12 = 0.6667% per month

Solve for PV?

PV = 1,145,189 (nearest ringgit)

The difference is a significant 399,995.

But the benefit of adjusting for inflation is that every year your income will increase by 4%, the rate of inflation. So in the second year the annual income will increase by 4% to 112,320. And in the third year the income will increase by another 4% to 116,813. To provide for this increase the target retirement fund has been increased by 399,995. It is good to get the client to target for the higher sum as inflation is a realistic scenario and prices of goods and services will increase and perhaps at an even larger degree than the 4% assumed.

Payments Growing by a Constant Percentage.

Just like in the inflation adjusted interest rate we can employ the same technique for a situation that will have a constant growth percentage in the payments. This will be experienced say in a savings plan that the participant says he will increase by a constant percentage so that he can reach his target better. Or he wishes to start small but start increasing his payment as his income or position changes. In these calculations we adjust the investment return for the constant percentage increase.

Example: let us say a person intends to save 10,000 a year for the first year but will increase this payment by 10% each year. If his investment return is 8% per annum how much will he have in his account at the end of 15 years?

For problems using adjusted interest rates the payment mode should be beginning mode. You can adjust the final figure if you want to adjust for an end mode payment. But the initial calculations must be in beginning mode. Let us input the known variables.

Payment = 10,000 (beginning mode)

N= 15.

$i = 8\% - 10\% / (1 + 0.1) = -1.8182$

Compute Present Value?

PV = 171,095

We now convert this Present Value into Future Value using the normal investment rate.

Therefore, PV = 171,095

N=15

i=8%

Compute Future Value?

FV = 542,742

If after finding the answer you want to discount one period because the payments were in fact made at the end of the year you divide the lump sum by $(1+i)$. Therefore the FV of 542,742 / by 1.08 = 502,539. This would be the sum accumulated had you used end mode of saving. When you use adjusted rates you find the present value first and then only you convert it to future value.

Increasing EPF Contributions Example

It is common to estimate a constant growth percentage for finding the Lump sum accumulated in the EPF if the salary increase by a constant percentage. Using the current contributions we will assume a constant growth percentage and the current EPF investment rate.

Example

Let us say you earn RM 3,000 a month. Your EPF contributions for the year are $3,000 * 12 * 0.23 = 8,280$. We calculate the Present value of these payments increasing by a constant percentage of say 5%. EPF offers an investment rate of also 5%. To calculate how much you will have in EPF in 20 years time, we use the adjusted interest rate technique. The inputs are:

Payment (beginning mode) = 8,280

N= 20 years

Adjusted interest rate = $5\% - 5\% / (1.05) = 0\%$

To compute the Present Value?

PV = 165,600.

We now convert this to Future Value using the actual investment rate of 5%.

PV = 165,600 (negative sign)

N=20

i=5%

Future Value = 439,386.

This figure had used beginning payments, but EPF contributions are made at the end of the period.

So now we discount the answer by $(1+0.5)$.

We will have $439,386 / 1.05 = 418,462$.

Effect of Compounding Frequency.

You would have noticed that the higher the frequency of compounding, like monthly compounding instead of annual compounding, the higher the future value. This is simply because in compounding the interest earned in each period is added to the principal and the interest is calculated on this compound figure. Therefore monthly compounding will add each month's interest earned to the principal and therefore derive a higher value.

Financial consultants will like to calculate the effect of compounding frequency on the return earned or yield, the term used to call the annual return.

With the calculator this is very easily done.

In the HP 10BII there are two shift functions called the Nominal Interest and the Effective interest. If the frequency of compounding is monthly and the stated annual interest rate is 10%, the effective rate is found by entering the nominal rate and then pressing for effective interest. You would enter the number of payments first as 12 in the calculator.

Therefore for 10% nominal interest compounded monthly the effective interest rate is:

10.4713. By compounding monthly you have added to the annual interest another 0.4713%. If the compounding frequency is daily, in other words the interest earned every day is added to the next day principal the effective interest rate becomes: 10.5156. You have effectively increased your annual return by 0.5156%.

In the Casio FC 100 and 200 there is a "conversion" key. You enter the nominal interest rate and press the conversion key to get the effective return. You would have to state the compounding frequency.

This compounding frequency is especially important in the payment of your mortgage loan. In the bad old days the banks used annual rest. In other words the principal paid is only taken into account at the end of the year. That era is thankfully over. Today we use monthly rest and the principal paid each month is taken into account in the computation of mortgage payment.

Effective Interest Rate Formula

The effective interest rate can be calculated using the formula:

$$i_{\text{eff}} = [1 + i/m]^m - 1$$

The formula reads as: the effective interest rate = 1 + nominal interest rate divided by the frequency of compounding (m), to the power of (m) minus 1.

Today because of competition banks have even resorted to offering daily rest loans. We have heard about Mortgage power and the Mortgage one. These are good developments. Competition is healthy for the consumer.

Uneven Cash Flows

In TVM calculations we had assumed that the payments made and received are the same, even or constant cash flows. However this is not true in many cases.

For example in a real estate investment the cash flows are uneven. The down payment is made only once. The mortgage payment may be even. The rent received may be even but they change on a regular basis as tenancy agreements are renewed. There is a one time payment received when you sell the asset and pay back the principal loan outstanding with the bank. All these cash flows must be entered and a rate of return determined for the investment.

The calculation for uneven cash flows has been made easy with the financial calculator which has a special cash flow function (CFj). With this function the cash flows are entered separately one at a time and the Present Value is calculated using the nominal interest rate. For investments this function allows you to calculate the internal rate of return (IRR) or the Net Present Value (NPV).

In these calculations there is a period 0. Most investments will require an investment at the beginning of the period. Therefore each problem has to enter a beginning value. It may most probably be negative. The cash flows are entered as negative or positive values as they truly are. There is no need for a negative sign key function as in TVM problems. A variety of problems can be solved using this cash flow function.

To Calculate the Present Value of College Payments that is Uneven.

Let us assume that the college fees are structured unevenly and they are paid at the beginning of the year.

For example:

1 st year fees	10,000
2 nd year fees	15,000
3 rd year fees	20,000
4 th year fees	30,000

You know you can earn 8% return on your accumulated college fund. What is the Present value or target sum needed to fund this college education costs. You cannot use TVM calculations as the payments are uneven. Therefore you have to use the cash flow (CFj) function key. What is the accumulated fund required to make these payments to the college.

In the HP 10BII you will enter the cash flows as follows: the amount can be entered as either negative or positive values because the education fund can be seen as a positive sum paying out the annual college fees.

You will state that there is one payment per year.

The 1st payment in time 0 (beginning of year one) is 10,000. You enter 10,000 and press CFj. You then enter 15,000 and press CFj and followed by 20,000 and 30,000. After that you enter the nominal interest rate that you can earn, in this case 8% and press i/per year. You then press the shift key and press for NPV. You will get the answer of 64,850.

Therefore with 64,850 earning 8% per year, you can pay out the four payments. The total payments add up to 75,000. But because of the investment return you only need 64,850 to fund the college education.

In the Casio FC 100 and 200 you have a special function key called “cash’ mode. You will enter the data according to a data field that appears. You then enter the interest and solve for Net Present Value.

Investment Appraisal

In the investment arena you will first have to invest some money before you can earn any returns. All these cash flows both negative and positive can be entered into the Cash flow function keys to evaluate the investment. There are two common measures, the Net Present Value and the Internal Rate of Return. These are called the Discounted Cash Flow (DCF) techniques. In other words all the cash flows are discounted to present values because these cash flows occur at different times and they have to be brought to present values and then compared. A positive net present value represents a good investment as you have positive cash flows even after using a required rate of return to discount the cash flows.

Net Present Value

The net present value of an investment is defined as the present value of the stream of cash inflows minus the present value of the stream of cash outflows, with both present values calculated on the basis of an appropriate rate of interest. Often this discount rate for the cash flows is the minimum required rate of return by the investor, or the cost of capital to the investor. The larger the positive net present value the better the investment return.

Internal Rate of Return

The internal rate of return on an investment is the interest rate that equates the present value of the stream of cash inflows to the present value of the stream of cash outflows. If this internal rate is larger than your required rate of return then the project or investment is accepted.

Investment appraisal problems are not part of the Module 1 syllabus but an example will help understand the application. With financial calculators the solution is quite straightforward.

Investment Appraisal Example - Properties

Property investment

Let us examine the cash flows of a property investment.

The cash flows are as follows:

Down payment plus expenses = 20,000

Bank loan = RM 200,000, 20 years, 8% interest monthly rest.

Property rented out for RM 1,800 per month for the first 5 years. At the end of 5 years you sell the property for RM 350,000 net of all expenses. Calculate the net present value and the internal rate of return of this property investment?

The first calculation is to find the monthly mortgage payment.

$PV = 200,000$

$N = 20 * 12 = 240$

$i = 8\% \text{ p.a.} / 12$

Solve for payment (end mode)?

Payment = 1,673 (nearest ringgit)

You will also have to calculate the bank loan outstanding at the end of 5 years. Using the amortization key you solve for balance outstanding.

The calculator shows the principal outstanding as RM 175,051.

You can now arrange all the cash flows:

Beginning cash flow [CF0] will be the down payment of RM 20,000 (negative).

At the end of the first year you will have total mortgage payments of RM 20,076. But you will also have collected rent of 1,500 per month. Total rent for the year = 18,000. The net cash flow for the first year excluding down payment is $18,000 - 20,076 = -2,076$

This negative cash flow of 2,076 will continue for the next three years, in total 4 years. In the fifth year the cash flow will be: the sale price less the bank loan repaid. It would be $RM\ 350,000 - 175,051 = +174,949$ and the difference between the mortgage payment and rent received of -2,076.

We now enter the cash flows in the CFj function.

$CF_0 = -20,000$

$CF_{j1} = -2,076$

$CF_{j2} = -2,076$

$CF_{j3} = -2,076$

$CF_{j4} = -2,076$

$CF_{j5} = -2,076 + 174,949 = +172,873$

You have to enter the required rate of return. Let us say it is 10%.

The Net Present Value = +80,759

The Internal rate of Return = 49.21%. This internal rate of return is superior to your required rate of return of 10%. In fact it is a fabulous investment.

If you had RM 100,000 you would have wished to have bought 5 houses at the same location. But that is not encouraged at all as there are numerous risks involved with property investment. To name a few: the availability of a tenant who will pay rent quite close to the mortgage payment. If your property remains untenanted you will have to bear the mortgage payments. Secondly the sales price is uncertain. This happens to be in a good location and therefore was able to fetch a good price. If you suddenly lose your income and are unable to pay your mortgage the banks will foreclose and you lose almost everything. Therefore there must always be a safety margin. Provide for at least six months of mortgage payments. Of course if you lose your income or if there are no tenants for more than six months you may have to liquidate your position. Call it a misadventure.

Self Assessment

1. Compute the future value of RM 100,000 if it is parked in an investment account earning 8.88% per annum compounded on half-yearly basis for 5 years?
 - a. RM 154,408
 - b. RM 153,017
 - c. RM 234,143
 - d. None of the above

2. How long will it take RM 100,000 to become RM1 million if it is allowed to grow at 10% per annum?
 - a. 24.16 years
 - b. 7.27 years
 - c. 15 25 years
 - d. None of the above

3. A unit trust fund was launched at par value of RM1 five years ago. Its current NAV is RM1.48. What is the compounded rate of return?
 - a. 9.6%
 - b. 8.03%
 - c. 8.16%
 - d. None of the above

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4. RM 250,000 was placed in a Bank fixed deposit account that generated interest monthly. All interests earned were ploughed back to earn interest at the same rate of return. What should be the nominal annual rate of return if the value had increased to RM 700,000 after 20 years?
- a. 5.16%
 - b. 5.28%
 - c. 14%
 - d. None of the above
5. Benny invested RM10,000 per annum for 20 years at the beginning of each and every year. If the return on investment was 7.75% per annum, how much should be the value of the investment after 20 years?
- a. RM 100,000
 - b. RM 44,500
 - c. RM 445,140
 - d. RM 479,640
6. Charles deposited RM 1,000 per month at the beginning of each and every month in an investment account for 10 years. The fund paid a return of 5.35% per annum on a monthly compounding basis. How much is available in the account at the end of 10 years?
- a. RM 158,930
 - b. RM 158,225
 - c. RM 153,423
 - d. None of the above
7. Julie required RM 500,000 for her daughter's education fund 15 years from now. She has been advised to place her regular annual deposit of same amount in an investment vehicle with a return of 6% per annum. How much must she set aside every year if the plan is to start immediately?
- a. RM 21,483
 - b. RM 20,265
 - c. RM 22,770
 - d. None of the above

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8. Mandy has a financial goal of building up RM 2 million for her retirement 10 years from today. She has RM 600,000 in existing financial assets that can earn yearly compounding return of 7.8% per annum. How much more must she invest every month, starting without delay if the vehicle chosen can yield annual return of 6% on a monthly compounding basis?
- RM 4,445
 - RM 4,423
 - RM 6,623
 - None of the above
9. Jack paid RM 15,000 every year for a 20-year life insurance endowment plan with a maturity value of RM 600,000. Ignoring the cost of insurance and assuming that there is no claim whatsoever, what is the yield of the policy?
- 6.77%
 - 6.22%
 - 6.34%
 - None of the above
10. It is Christmas time and Johnson has just received his bonus. From his annual bonus, Johnson plans to set aside 10% of his salary every year for investment in an account for retirement purpose. He is currently earning RM 80,000 every year and his salary is expected to increase by 5% per year until Johnson retires 15 years later. Could you advise Johnson how much he will have as retirement fund if the return on investment is 6.25% on the assumption that the plan takes effect immediately?
- RM 2,746,031
 - RM 274,603
 - RM 258,450
 - None of the above

Answers: 1- A, 2.-A, 3-C, 4.-A, 5-D, 6-A, 7-B, 8-B, 9-B, 10-B.

